Modeling Long-Term Sustainability

Kaoru Yamaguchi *

1 Introduction

In the book *Beyond Walras, Keynes and Marx* [8] , I have tried to synthesize three effete economic paradigms in the industrial age: neoclassical, Keynesian, and Marxian schools, under a general equilibrium framework, and presented a new economic paradigm suitable for the information age. Through the work, my interest has gradually shifted toward a sustainability of such new economy. This shift of interest resulted in the edition of the book *Sustainable Global Communities in the Information Age* [9], in which I have contributed a chapter myself: *Sustainability and a MuRatopian Economy.* In the chapter, sustainability is newly defined in terms of physical, social and ecological *reproducibility* from a general equilibrium point of view developed in [8]. At that time, I had no tools or softwares available with me which enable to model my framework of sustainability for further computational analysis and simulation.

Soon after, I happened to encounter the book *Beyond the Limit* [3]. From the book, I learned the existence of the STELLA software which constructed the World3 model on Macintosh. I was amazed by its capability of being able to build complicated models easily such as the World3 model presented in the appendix of the book, and gradually became interested in the software which can handle complex socio-economic dynamics without knowing computer languages such as C and C^{++}. The World3 model, I also learned from the book, turned out to be an extension of the World Model originally developed by Jay Forrester in his book *World Dynamics* [5]. The model was created by a computer software called DYNAMO Compiler. Such dynamic modeling methodology, which had been developed by Jay Forrester in 1960s at the Sloan School of Management, MIT, is now widely called *System Dynamics (SD)* .

Mathematically speaking, system dynamics is a modeling algorism and method by which dynamical systems of difference and differential equations are numerically solved and solutions are easily visualized by a computer so that further

*The author is professor at the Graduate School of Business, Doshisha University, Kyoto, Japan. He's contacted at kaoyamag@mail.doshisha.ac.jp. This paper is partly based on his paper: A Step-By-Step System Dynamics Modeling Of Sustainability, submitted at the 19th International Conference of the System Dynamics Society, Atlanta, Georgia, USA, July 23 – 27, 2001. A slightly simplified version of this paper is contributed to the chaper 3 of "Handbook of Sustainable Development Planning: Studies in Modeling and Decision Support" ed. by M. Quaddus and A. Siddique, Edward Elgar Publishers, U.K. 2004.

analytical simulations are made possible. In this sense, SD can cover many dynamic fields such as physics, chemistry, medicine, biology, ecology, business management, economics, environment, etc.

Specifically, this turns out to be a good tool for social scientists, consultants and business managers whose specialities are not computational sciences and mathematics, but need dynamic modeling for the understanding of a complex real world, because modeling dynamical systems becomes as easy as drawing a picture on a canvas. Several softwares of system dynamics have been available, running on personal computers, such as Dynamo, Stella, Vensim and Powersim. Brief summaries of these four softwares are found in the Appendices of the book *Modeling the Environment* [4]. The reader can also find more specific software information easily on the Internet.

It was indeed fortunate for me to get familiarized with system dynamics. Accordingly, using Vensim software in this paper, I was able to build a system dynamics model of sustainability based on my framework discussed above. To be specific, a macroeconomic growth model is employed as a starting point. Then a meaning of sustainability is clarified by expanding the model step by step from a simple macroeconomic growth model to a complicated ecological model. In the due course sustainability is represented in terms of physical, social and ecological reproducibilities. As an implementation of the analysis, it is shown that a continued economic growth is unsustainable in the long run as long as non-renewable resources are needed.

To make our modeling a little bit more analytical, a step-by-step procedure of system dynamics modeling is developed from a viewpoint of a mathematical system of difference equations. Through this procedure, essential concepts for building a SD model would be better understood, such as the difference between a moment and a period of time, a unit check, a computational procedure for feedback loops, an expansion of boundaries, and a limit to an analytical mathematical model. Accordingly, at the end of the paper the reader will become familiarized with a basic concept of system dynamics as well as sustainability.

The original draft was mostly written while I was visiting the Hawaii Research Center for Futures Studies, University of Hawaii at Manoa in March 2001, at the invitation of its director, Dr. James Dator. Hawaii turned out to be a good place for me to deeply consider sustainability. Since Hawaii became the 50th US state in 1959, only less than a half century has passed. Yet, its economy with rapidly increased 1.2 million islanders needs more than five million tourists and their food annually for its survival, while dumping as much garbage in its small islands. Can Hawaii be sustainable for the 21st century and beyond? Why did the Easter Island in the southern Pacific Ocean suddenly collapse: overpopulation, lack of food, water and natural resources, wars, epidemics ? The following sustainability model is developed with these naive questions in mind.

Table 1: Unknown Variables and Constants (1)

Category	Symbol	Notation	Unit
Unknown Variables	K_{t+1}	Capital Stock	machine
	Y_t	Output (or Income)	food/year
	C_t	Consumption	food/year
	S_t	Saving	food/year
	I_t	Investment	machine/year
Constants	v	Capital-Output Ratio (= 4)	machine/(food/year)
	c	Marginal Propensity to Consume (= 0.8)	dimensionless
Initial value	K_t	Initial Capital Stock (=400)	machine

2 A Macroeconomic Growth Model

A step-by-step modeling of sustainability starts with a simple macroeconomic growth model which can be found in macroeconomic textbooks. It consists of the following five equations.

$$K_{t+1} = K_t + I_t \qquad \text{(Capital Accumulation)} \tag{1}$$

$$Y_t = \frac{1}{v}K_t \qquad \text{(Production Function)} \tag{2}$$

$$C_t = cY_t \qquad \text{(Consumption Function)} \tag{3}$$

$$S_t = Y_t - C_t \qquad \text{(Saving Function)} \tag{4}$$

$$I_t = S_t \qquad \text{(Equilibrium Condition)} \tag{5}$$

Equation (1) represents a capital accumulation process in which capital stock is increased by the amount of investment. Output is assumed to be produced only by capital stock in a macroeconomic production function (2). The amount of consumption is assumed to be a portion of output - a well-known macroeconomic consumption function (3). Saving is defined as the amount of output less consumption in (4). At the equilibrium investment has to be equal to saving as shown in (5), otherwise output would not be sold out completely or in a state of shortage.

These five equations become simple enough to describe a macroeconomic growth process. Most of the symbols used in the above equations should be familiar for economics and business students. Precise meaning of these variables, however, are usually left unexplained in the textbooks. SD modeling, on the other hand, requires precise specification of their units, as defined in Table 1, without which it is impossible to construct a model. It is thus worth considering these specifications in detail.

Time

As emphasized in [10], it is fundamental in SD modeling to make a distinction between two different concepts of time. One concept is to represent time as a moment of time or a point in time, denoted here by τ; that is, time is depicted as a real number such that τ= 1, 2, 3, ... It is used to define the amount of stock at a specific moment in time. The other concept is to represent time as a period of time or an interval of time, denoted here by t, such that t = 1st, 2nd, 3rd, ..., or more loosely t = 1, 2, 3, ... It is used to denote the amount of flow during a specific period of time. Units of the period could be a second, a minute, an hour, a week, a month, a quarter, a year, a decade, a century, a millennium, etc., depending on the nature of the dynamics in question. In a macroeconomic analysis, a year is usually taken as a unit period of time.

With these distinctions in mind, the equation of capital accumulation (1), consisting of a stock of capital and a flow of investment, has to be precisely described as

$$K_{\tau+1} = K_{\tau} + I_t \qquad \tau \text{ and } t = 2001, 2002, 2003, \cdots \tag{6}$$

A confusion, however, might arise from these dual notations of time, τ and t, no matter how precise they are. It would be better if we could describe stock-flow relation uniformly in terms of either one of these two concepts of time. Which one should, then, be adopted? A point in time τ could be interpreted as a limit point of the interval of time t. Hence, t can portray both concepts adequately, and usually be chosen.

When t is used to represent a unit interval between τ and $\tau+1$, the amount of stock K_t thus defined at the t-th interval could be interpreted as an amount at the beginning point τ of a period t or the ending point $\tau+1$ of the period t; that is,

$$K_t = K_{\tau} \qquad \text{Beginning amount of stock} \tag{7}$$

or

$$K_t = K_{\tau+1} \qquad \text{Ending amount of stock} \tag{8}$$

When the beginning amount of the stock equation (7) is applied, a stock-flow equation of capital accumulation (6) is rewritten as

$$K_{t+1} = K_t + I_t \qquad t = 2001, 2002, 2003, \cdots \tag{9}$$

In this formula, capital stock K_{t+1} is evaluated at the beginning of the period $t+1$; that is, a flow of investment I_t is to be added to the present stock value of K_t for the evaluation of the capital stock at the next period.

When the ending amount of the capital stock equation (8) is applied, the stock-flow equation (6) is rewritten as

$$K_t = K_{t-1} + I_t \qquad t = 2001, 2002, 2003, \cdots \tag{10}$$

Two different concepts of time - a point in time and a period of time - are in this way successfully unified. It is very important for the beginners of SD modeling to understand that time in system dynamics usually implies a period of time which has a unit interval. Periods need not be discrete and could be continuous. The beginning amount of capital accumulation (9) is employed here as many macroeconomic textbooks do[1].

Unit

In SD modeling, units of all variables, whether unknowns or constants, have to be explicitly declared. In equation (5) of equilibrium condition,

investment is defined as an amount of machine per year, while saving is measured by an amount of food per year. Therefore, in order to make the equation (5) congruous, a unit conversion factor ξ of a unitary value has to be multiplied such that

$$I_t = S_t * \xi, \tag{13}$$

in which ξ converts a food unit of saving to a machine unit of capital investment; that is to say, it has a unit of machine/food dimension. This tedious procedure of unit conversion could be circumvented by replacing machine and food units with a dollar unit as many macroeconomic textbooks implicitly presume so.

Model Consistency

A model consistency has to be examined as a next step in SD modeling, following time and a unit check. A model is said to be at least consistent if it has the same number of equations and unknown variables. This is a minimum requirement for any model to be consistent. The above macroeconomic growth model consists of five equations with five unknowns and two constants. Thus, it becomes consistent.

Let us now consider how these equations are computationally solved. Starting with the initial condition of the capital stock K_t, numerical values are assigned from the right-hand variable to the left-hand variables as follows:

$$K_t \to Y_t \to C_t \to S_t \to I_t \to K_{t+1} \tag{14}$$

This is how computer can numerically solve the equations of our dynamic macroeconomic model.

[1]To show the difference between stock and flow explicitly, it would be informative to decompose the capital accumulation equation (1) as follows:

$$K_{t+1} \equiv K_t + \Delta K_t \quad \text{(Identity of Capital Stock Accumulation)} \tag{11}$$

$$\Delta K_t = I_t \quad \text{(Investment as a Flow of Capital)} \tag{12}$$

Feedback Loop

There are two types of equations in our macroeconomic growth model. One type is the equation of *stock-flow relation* which specifies a dynamic movement. Capital accumulation equation (1) is of this type. The other type is the equation of *causal relation* in which a left-hand variable is caused by right-hand variables (and constants). The remaining four equations in the macroeconomic growth model are of this type.

These two types are clearly distinguished in SD modeling as illustrated in Figure 1. A stock-flow relation is illustrated by a rectangular box that is connected by a double-lined arrow with a flow-regulating faucet, while a causal relation is drawn by a single-lined arrow. Then, we can easily trace a loop of arrows starting from a rectangular box and coming back to the same box. Such a loop is called *a feedback loop* in system dynamics. A feedback loop, in this way, has to include at least one stock-flow equation. Simultaneous equation system, on the other hand, has only equations of causal relation and, accordingly, cannot have feedback loops. Without a feedback loop, system cannot be dynamic.

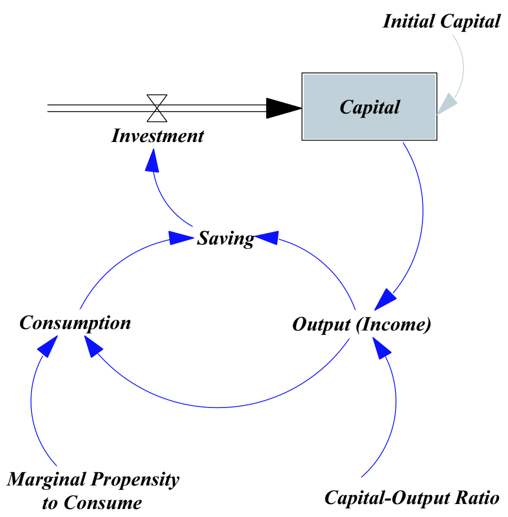

Figure 1: Simple Macroeconomic Growth Model

A feedback loop is called *positive* if an increase in stock results in an increase in a coming back stock, while *negative* if a decreased amount of coming back stock results in. There are two feedback loops starting from the capital stock box in our macroeconomic model, and we can easily show that one loop is positive and the other is negative. Only a negative feedback loop corresponds with a computational trace of the equation (14). In this sense, SD diagram can be said be more powerful tool than a system of equations for identifying causal loops.

A Steady State Equilibrium

Since SD modeling is by its nature dynamic, it is very important to find out a steady state equilibrium for a structural consistency of the model. Steady state implies that all stocks stop changing, which in turn means that the amount of flows (to be precise, net flows) becomes zero. In other words, it is a state of no growth. In our mode a steady state equilibrium of capital accumulation is attained for $K_{t+1} = K_t$. To calculate the steady state analytically, five equations of the model have to be first reduced to a single equation of capital accumulation:

$$K_{t+1} = \left(1 + \frac{1-c}{v}\right) K_t. \tag{15}$$

Then, a steady state is easily shown to exit for $c = 1$; that is, the amount of output is all consumed and no saving is made available for investment. In our numerical example, a steady state equilibrium is attained at the values of $K^* = 400, Y^* = C^* = 100$, and $S^* = I^* = 0$.

Simulation for an Economic Growth

Let us try to drive the economy out of this steady state equilibrium. A growth path can be easily found by setting "a marginal propensity to consume" to be less than unitary; say, $c = 0.8$. Then 20% of output (or income) is saved for investment, which in turn increases the capital stock by the amount of 20, which then contributes to the increase in output by 5 next period, driving the economy toward an indefinite growth. Table 2 shows how capital, output, consumption and investment grow at a growth rate of 5% for $c = 0.8$.

Let us consider another growth path in which maximum amount of saving is made first at the cost of consumption, then, by accumulating capital stock as fast as possible, a higher level of consumption is enjoyed later. This type of growth path can be built by making "a marginal propensity to consume" as a function of a normalized output level such that

$$c = c(Y_t/Y_{norm}) \tag{16}$$

where Y_{norm} is a normalized reference level of output with which a current level of output is compared. Usually an initial value of output is selected as a reference level: $Y_{norm} = Y_{initial} = 100$. This function is called a table function, or a graphic function, or a lookup function in SD modeling.

A simple example is the following linear function as illustrated in Figure 2:

$$c = 0.4\,\frac{Y_t}{Y_{initial}} + 0.2 \tag{17}$$

Table 2: Macroeconomic Growth Model

Year	Capital	Output	Consumption	Investment
2001	400.00	100.00	80.00	20.00
2002	420.00	105.00	84.00	21.00
2003	441.00	110.25	88.20	22.04
2004	463.04	115.76	92.61	23.15
2005	468.20	121.55	97.24	24.31
2006	510.51	127.62	102.10	25.52
2007	536.03	134.00	107.20	26.80
2008	562.84	140.71	112.56	28.14
2009	590.98	147.74	118.19	29.54
2010	620.53	155.13	124.10	31.02

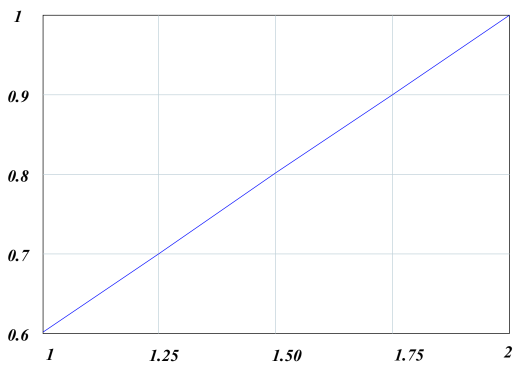

Figure 2: A Table Function of A Marginal Propensity to Consume

In the beginning "marginal propensity to consume" is set to be the lowest (or a subsistence) level, say, $c(1) = 0.6$, to allow for a maximum growth rate, then it gradually becomes higher as income increases, enabling more consumption. When income level doubles, we have $c(2) = 1$ and no further saving and investment are made; that is, a maximum consumption level is enjoyed. Figure 3 illustrates a gradual increase in the value of "marginal propensity to consume" and a gradual decrease in the growth rate.

Building up a table function implies connecting the variable *Output* by a single-lined arrow to the constant *Marginal Propensity to Consume* in Figure 1. And an exogenous constant of "marginal propensity to consume" which has been residing outside the model now becomes an endogenous unknown variable whose value is to be determined by the behavior of the model itself. Better modeling is to reduce the number of constants and make a model self-determined by itself without relying on the exogenous values of constants. In this sense, a

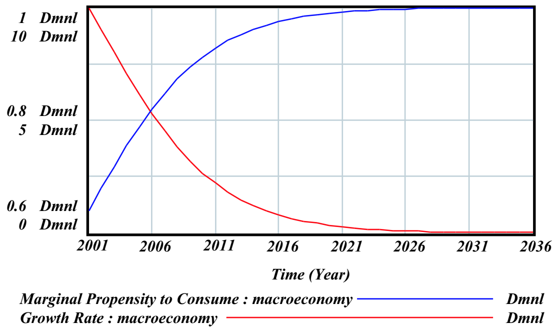

Figure 3: Growth Rate and Marginal Propensity to Consume

capability of introducing table functions is one of the most powerful features in SD modeling. In fact, an introduction of nonlinear and/or numerical table functions enables many diversified dynamic behaviors for analytical simulations.

DISCUSSIONS FOR THE SECTION:

(a - i) Identify two feedback loops in Figure 1, and tell which is positive or negative.
(a - ii) Write down Vensim equations of the Simple Macroeconomic Growth Model by referencing to the equations (1) - (5) and those in the Appendix. By doing so, you will be able to identify two types of equations discussed above more clearly. *Stock-flow relation* is generically written in Vensim as

```
Stock = INTEG (Inflow - Outflow, Initial Value)
```

(b) Derive the equation (15). Then consider the meaning of a steady state equilibrium.

(c) What will happen if a consumption level drops and "marginal propensity to consume" becomes, say, $c = 0.7$. Calculate new levels of Capital, Output, Consumption and Investment, and fill in the columns in Table 2.

3 Physical Reproducibility

Sustainability

In the above macroeconomic growth model, depreciation of capital stock is not considered, or I_t is regarded as net investment. In reality, capital stocks depreciate, and, for maintaining the current level of output, some portion of the income has to be saved to replace the depreciation. When a depreciation rate is high, a higher portion of income has to be saved at the cost of the consumption. Here arises a sustainability issue of the economy: how to maintain a level of income for sustainable development. In this sense a sustainability issue has been as old as human history.

After the UN Conference on Environment and Development (UNCED), widely known as the *Earth Summit*, in Rio de Janeiro, Brazil, 1992, *sustainable development* becomes a fashionable word in our daily conversations. This might be an indication that our awareness on environmental crises such as global warming, acid rain, depletion of the ozone layer, tropical deforestation, desertification, and endangered species has deepened. How should, then, a state of sustainable development be defined? Some proposed definitions are the following:

> Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. [7, p.43].

> The simplest definition is: A sustainable society is one that can persist over generations, one that is far-seeing enough, flexible enough, and wise enough not to undermine either its *physical* or its *social* systems of supports. (*Italic* emphases made by the author) [3, p.209].

These definitions are articulated so as to be understood even by children. However, from an economist's point of view, these definitions lack an interrelated view of production, consumption, society and environment.

Sustainability is comprehensively defined when all activities in economy, society and nature are interpreted as reproduction processes; that is, in terms of physical, social and ecological reproducibility [9]. A merit to this approach is that an economic structure of reproduction processes such as constructed in the general equilibrium framework [8] can be applied, since the most basic activity in any society is a reproduction process in which inputs are repeatedly transformed into outputs for consumption and investment each year. Hence, sustainability is similarly presented here as an economic process of physical, social and ecological reproduction step-by-step. In this way the interrelationship between economic activities and environment is integrated wholistically.

Capital Depreciation

Let us introduce depreciation in the macroeconomic growth model. The equation of capital accumulation (1) is expanded as follows:

$$K_{t+1} = K_t + I_t - D_t \qquad \text{(Capital Accumulation)} \qquad (18)$$

$$D_t = \delta K_t \qquad \text{(Capital Depreciation)} \qquad (19)$$

As Figure 4 shows, this can be easily done in SD modeling by adding an outflow arrow of depreciation from the capital stock. I_t in equation (18) is now reinterpreted as gross investment.

Table 3: Unknown Variable and Constant Added (2)

New Variable	D_t	Depreciation	machine/year
New Constant	δ	Depreciation Rate (=0.02)	1/year

Physical reproducibility implies that gross investment is greater than or equal to the depreciation.

$$I_t - D_t \geq 0 \qquad \text{[Physical Reproducibility]} \tag{20}$$

The macroeconomic growth model with depreciation, which is here called physical reproducibility model, now consists of six equations with six unknown variables: $K_{t+1}, Y_t, C_t, S_t, I_t, D_t$ and three constants: v, c, δ .

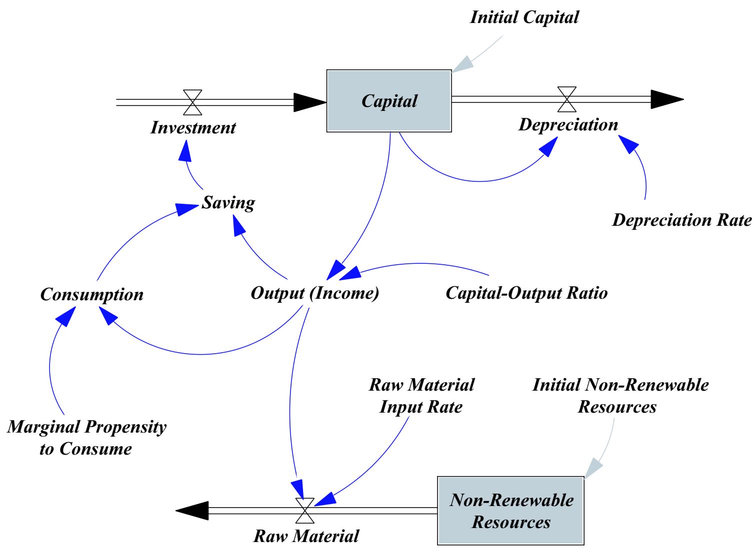

Figure 4: Physical Reproducibility Model

A Steady State Equilibrium

A steady state equilibrium is attained at $K_{t+1} = K_t$ or $I_t = D_t$, as easily shown from the equation (18). To obtain the steady state analytically, all equations in the model have to be reduced to a single capital accumulation equation:

$$K_{t+1} = \left(1 + \frac{1-c}{v} - \delta\right) K_t. \tag{21}$$

A steady state condition is then easily obtained as follows (asterisks are added to the constants that meet this condition):

$$\frac{1-c^*}{v^*} = \delta^* \tag{22}$$

At the steady state, "marginal propensity to consume" becomes less than unitary; $c^* = 1 - \delta^* v^* < 1$, which implies that a portion of output has to be saved to replace the capital depreciation. One possible combination of numerical values for the steady state is $(v^*, c^*, \delta^*) = (4, 0.8, 0.05)$.

Simulation for an Economic Growth

For the economy to grow out of the steady state; that is, $K_{t+1} > K_t$, at least one of the following three actions has to be taken.

(1) Increase productivity $(\frac{1}{v} > \frac{1}{v^*})$ or $v < v^*$.

(2) Decrease consumption (or increase saving and investment) $c < c^*$.

(3) Improve capital maintenance $\delta < \delta^*$.

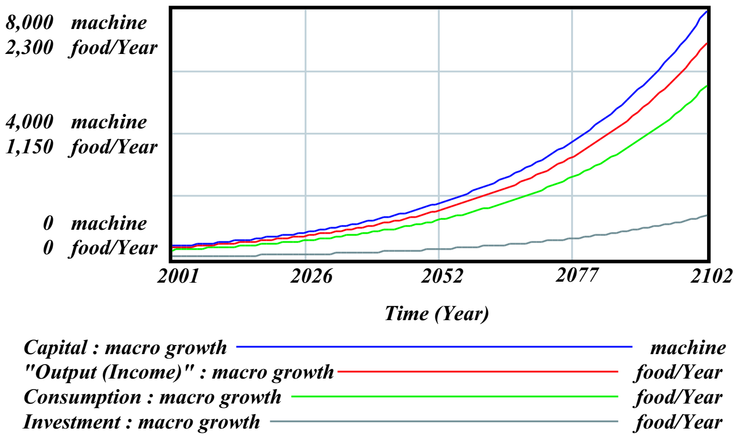

Figure 5: A Simulation for Economic Growth

As one such numerical example, let us take the case (3) and reset the rate of depreciation at $\delta = 0.02$ from $\delta^* = 0.05$. In this case, an economic growth becomes 3%. As Figure 5 illustrates, during the 21st century capital stock keeps increasing from $K_{2001} = 400$ to $K_{2101} = 7,687$ and so does output from $Y_{2001} = 100$ to $Y_{2101} = 1,921$, more than 19 folds! Can such a growth be sustainable?

Non-Renewable Resource Availability

The physical reproducibility condition (20) presupposes an availability of non-renewable natural resources which is represented by the following equation:

$$R_{t+1} \equiv R_t - \Delta R_t \qquad \text{(Non-Renewable Resource Depletion)} \qquad (23)$$

$$\Delta R_t = \lambda Y_t \qquad \text{(Non-Renewable Raw Material Input)} \qquad (24)$$

For simplicity, let us assume that non-renewable resources are represented by fossil fuels such as coal, gas, and oil whose units are uniformly measured by a ton. Then, λ is interpreted as an input amount of fossil fuels necessary for producing a unit of output.

Table 4: Unknown Variable and Constant Added (3)

New Variable	R_{t+1}	Non-Renewable Resource	ton
New Constant	λ	Raw Material Input Rate (=0.05)	ton/ food
Initial value	R_t	Initial Non-Renewable Resource (=1,00)	ton

Assuming that equations (23) and (24) are reduced to one equation, we have now seven equations for seven unknown variables and four constants. Hence the model is shown to be consistent.

Let us next consider the existence of a steady state equilibrium. There are two state variables K_{t+1} and R_{t+1} in the model. A steady state of capital accumulation is not affected by the introduction of non-renewable resources, while a steady state of non-renewable resources implies $R_{t+1} = R_t$, which in turn means $\Delta R_t = \lambda Y_t = 0$ or $Y_t = 0$. On the other hand, a steady state equilibrium of capital stock implies a positive amount of output; that is, $Y_t > 0$. A contradiction arises! Hence it is concluded that a macroeconomic growth model with non-renewable resources cannot have a steady state equilibrium by its nature. To make the model feasible, the existence of a steady state equilibrium of non-renewable resources has to be conceptually given up. Or, non-renewable natural resources have to be assumed to be available at any time in the economy so that the earth's limited source of non-renewable resources is not depleted; that is,

$$\sum_{t=2001}^{\infty} \Delta R_t < R_{2001} \qquad \text{[Non-Renewable Resource Availability]} \qquad (25)$$

Simulation for Sustainability

Non-renewable resources are continuously deleted even at a steady state equilibrium of capital accumulation, contrary to a general belief that they are not in a non-growing economy.

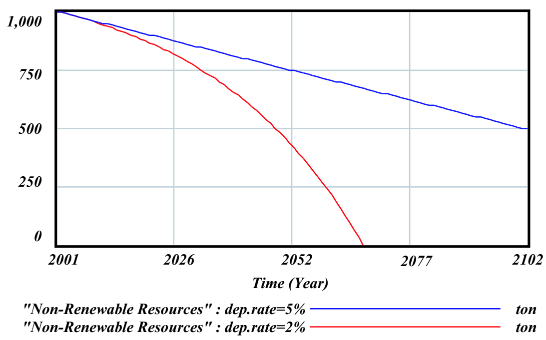

Figure 6: Depletion of Non-Renewable Resources

At the steady state equilibrium set by the condition (22), as Figure 6 illustrates, the initial non-renewable resources $R_{2001} = 1,000$ constantly diminishes by one half a century later; that is, $R_{2101} = 500$. This can be easily examined by a simple calculation. Since the economy is at the steady state, the output level remains constant at $Y_t = 100$. Hence, $\Delta R_t = \lambda Y_t = 0.05 \cdot 100 = 5$ and non-renewable resources are depleted by five tons every year. Over a century they will be depleted by 500 tons. It is very important, therefore, to understand that *a steady state equilibrium is not sustainable in the long run.* In fact, a simple calculation shows that non-renewable resources will be totally exhausted over two centuries; that is, by the year 2201 we will have $R_{2201} = 0$.

To show how fast non-renewable resources deplete under a growing economy, a depreciation rate is reset again to $\delta = 0.02$ and the economy starts growing at the rate of 3%. In this case, non-renewable resources will be totally depleted in the year 2066; that is, at the beginning of the next year we will have $R_{2067} = -5.813$, as Figure 6 illustrates.

How can we circumvent such a faster depletion of non-renewable resources and stay within a limit to resource availability and physical reproducibility? First, an efficient use of non-renewable natural resources has to be invented. For this, an introduction of long-term management of resources will be necessary. Second, substitutes for non-renewable resources have to be discovered or newly invented through technological breakthroughs. For this, research and development of new technology have to be oriented toward this direction. The issue of substitutes for non-renewable resources will be more fully analyzed in the next section.

Feedback Loop for Non-Renewable Resource Availability

What will happen if the development of substitutes are delayed or failed? To overcome a diminishing non-renewable resources, two self-regulating forces might appear in the economy. The first and more direct force is to curb down a raw material input rate λ. In a market economy, this might emerge as an increase in prices of non-renewable resources so that their use will be regulated. In SD modeling, this self-regulating force can be easily implemented by drawing an arrow from a stock of non-renewable resources to a constant of the raw material input rate λ and defining a table function as follows:

$$\lambda = \lambda \left(\frac{R_t}{R_{initial}} \right) \tag{26}$$

The second and more indirect force might appear as a reduction of productivity as non-renewable resources begin to be exhausted. In other words, a productivity which is defined as $\frac{1}{v}$ might begin to slide down. In SD modeling, this self-regulating force can be easily implemented by drawing an arrow from non-renewable resources to a capital-output ratio and defining a table function as follows:

$$v = v \left(\frac{R_t}{R_{initial}} \right) \tag{27}$$

The second force of self-regulation is considered here as an example of the effect of diminishing non-renewable resources on the economy. Let us assume that a productivity is not affected until non-renewable resources are depleted up to 40%. Then it begins to decrease as non-renewable resources continue to be depleted. Table 5 indicates one such numerical example of diminishing

Table 5: A Table Function of Capital-Output Ratio

$R_t/R_{initial}$	0	0.1	0.2	0.3	0.4	0.5	0.6 - 1
v	20	16	12	8	6	5	4

productivity (or an increasing capital-output ratio).

Figure 7 illustrates the effect of such self-regulating forces. Output level attains its highest peak in the year 2043; $Y_{2043} = 337.53$, then begins to decrease. In the year 2093, the output level becomes less than its initial output level; $Y_{2093} = 98.15 < Y_{2001} = 100$. Apparently at this lower level of output the initial number of population would not be sustained. In other words, non-renewable resource availability and population growth become a serious trade-off, and either the preservation of non-renewable resources or population growth has to be sacrificed in the long run. To see this trade-off relation of sustainability more explicitly, the equation of population growth has to be brought into the model, which will be done in the next section.

DISCUSSIONS FOR THE SECTION:

(a) Two concepts of sustainability are quoted in this section. Try to explain what is meant by these concepts in your own words, and see if your friends can understand your explanation.

(b) Three cases are identified for the economy to grow out of the steady state, and only the case (3): Improve capital maintenance, is considered in detail. Try

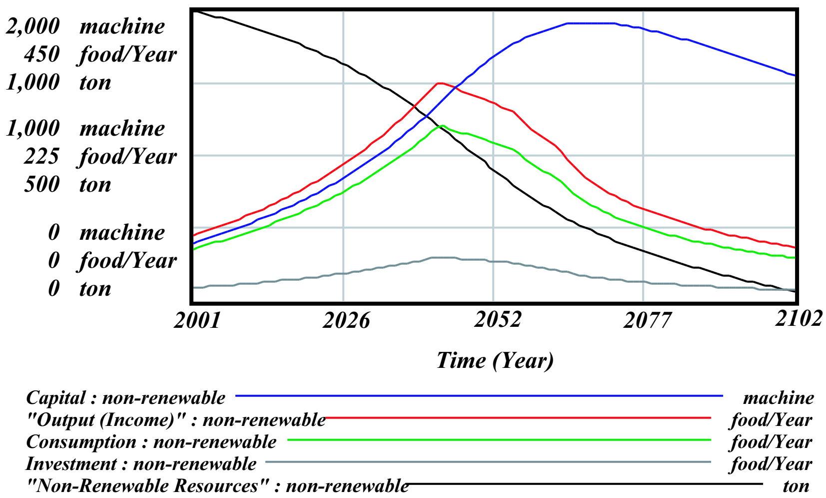

Figure 7: Physical Reproducibility with Non-Renewable Resources

to analyze the remaining two cases by drawing up-arrow or down-arrow for the variables that might be caused to be increased or decreased as constants v, c change in Figure 4: Physical Reproducibility Model.

(c) When non-renewable resources begin to be depleted, two possible feedback loops or self-regulating forces are pointed out to emerge. A direct force is to curb the raw material input rate of λ, while an indirect force is to influence a capital-output ratio v or productivity. Pick up some examples of non-renewable resources such as oil, and discuss what kind of self-regulating forces might appear in reality as such resources begin to be depleted.

4 Social Reproducibility

To consider social reproducibility as a next step of sustainability, population growth is now embodied in the model as follows:

$$N_{t+1} \equiv N_t + \Delta N_t \qquad \text{(Population Growth)} \qquad (28)$$

$$\Delta N_t = \alpha N_t - \beta N_t \qquad \text{(Net Birth = Birth - Death)} \qquad (29)$$

Notations of new variables and constants are shown in Table 6.

For a survival of any society a minimum amount of consumption has to be at least produced each period to reproduce its population. This amount needs not be a subsistence amount, but has to be enough "to maintain the minimum

Table 6: Unknown Variables and Constants Added (4)

New Variables	N_{t+1}	Population	person
	L_t	Workers (Labor Force)	person
New Constants	α	Birth Rate (= 0.03)	1/year
	β	Death Rate (= 0.01)	1/year
	θ	Participation Ratio to Labor Force(= 0.6)	dimensionless
	ℓ	Output-Labor Ratio (= 0.4)	(food/year)/person
	$\underline{c}$	Minimum Standards of Consumption (= 0.16)	(food/year)/person
Initial value	N_t	Initial Population (=500)	person

standards of *wholesome and cultured living* (Article 25, The Constitution of Japan)." Let $\underline{c}$ be such minimum standards of consumption per capita. Then, a total amount of consumption defined in the consumption function (3) has to be replaced with the following:

$$C_t = \underline{c}N_t \qquad \text{(Minimum Standards of Consumption)} \tag{30}$$

With the introduction of the minimum standards of consumption that is demanded irrespective of the output level, the amount of saving defined in the saving function (4) might become negative as population and consumption increase. To warrant a non-negative amount of saving, the saving function has to be technically revised as follows:

$$S_t = \text{Max}\{Y_t - C_t, 0\} \qquad \text{(Non-Negative Saving)} \tag{31}$$

Social reproducibility is now defined as a reproduction process in which minimum standards of consumption is always secured out of the net output[2]; that is,

$$Y_t - D_t - \underline{c}N_t \geq 0 \qquad \text{[Social Reproducibility]} \tag{33}$$

Note that whenever this social reproducibility condition is met, physical reproducibility (20) also holds; that is,

$$I_t = S_t = Y_t - C_t = Y_t - \underline{c}N_t \geq D_t \tag{34}$$

[2] To be precise, a unit of depreciation (machine/year) has to be converted to a unit of food (food/year) as follows:

$$Y_t - D_t/\xi \tag{32}$$

as in the equation (13)

With the introduction of population, the number of workers or labor force is easily defined as a portion of the population:

$$L_t = \theta N_t \qquad \text{(Workers)} \qquad (35)$$

Production function (2) is then replaced with the following revised production function which allows an inclusion of labor force explicitly as a new factor of production[3].

$$Y_t = \text{Min}\{\frac{1}{v}K_t,\ \ell L_t\} \qquad \text{(Production Function)} \qquad (37)$$

A Steady State Equilibrium

Our macroeconomic growth model is now getting a little bit complex. From the tables of unknown variables and constants (1) through (4), nine unknown variables and nine constants are enumerated for nine equations. Therefore, the model is shown to be consistent.

Let us now consider a steady state equilibrium. There are three variables of stocks such as capital, population and non-renewable resources; $K_{t+1}, N_{t+1}, R_{t+1}$. A steady state of population growth $N_{t+1} = N_t$ is attained when a birth rate is equal to a death rate; say, $\alpha^* = \beta^* = 0.01$. Meanwhile, as already mentioned above, no steady state equilibrium is possible for non-renewable resources R_{t+1}. As to a steady state of capital stock K_{t+1}, two cases of steady state equilibria may emerge due to the introduction of the new production function (37).

(1) A case in which output is constrained by capital stock: $Y_t = \frac{1}{v}K_t$. In this case, from a simple calculation we have

$$\frac{K_t}{N_t} = \frac{\underline{c}}{\frac{1}{v} - \delta} = 0.8 \qquad (38)$$

for a depreciation rate of $\delta = 0.05$. When $N_t = 500$, capital stock has to be $K_t = 400$ at the steady state. Hence, a steady state equilibrium is summarized as $(K^*, N^*, Y^*, C^*, S^*, I^*) = (400, 500, 100, 80, 20, 20)$, except that non-renewable resources keep depleting by the amount of five tons every year as analyzed under the previous section of physical reproducibility. When $K_t < K^*$, the capital stock continues to be reduced to zero, while if $K_t > K^*$, it tends to converge to 800 – another steady state to be discussed below. In this sense, this steady state is said to be *unstable* at a critical value of $K^* = 400$.

[3] Alternatively, a neoclassical production function such as a Cobb-Douglas production function can be used without any difficulty in SD modeling as follows:

$$Y_t = AK_t^a L_t^{1-a} \qquad (36)$$

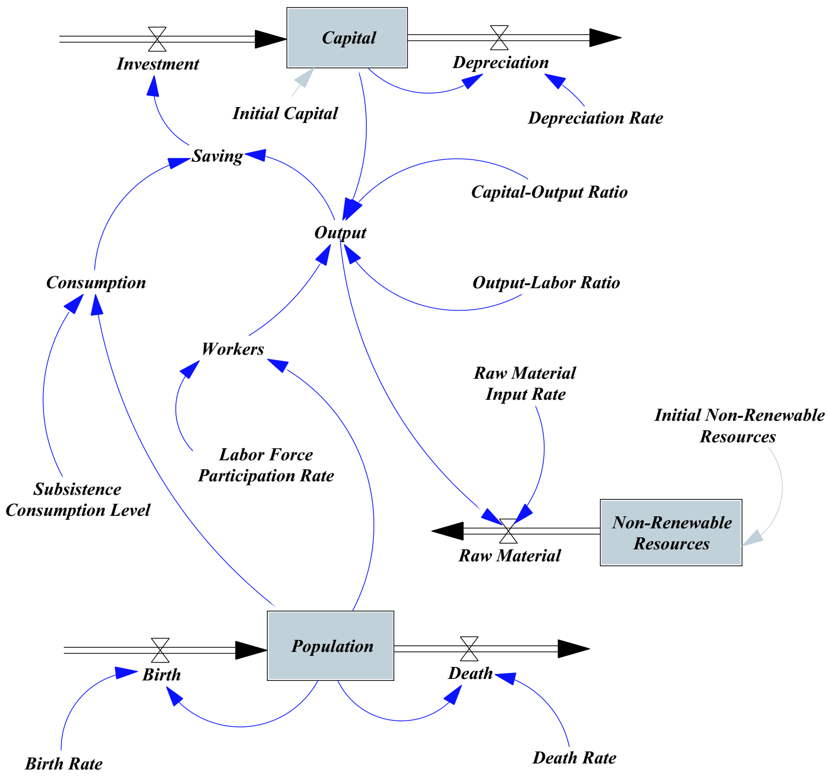

Figure 8: Social Reproducibility Model

(2) A case in which output is constrained by workers: $Y_t = \ell L_t$. In this case, we have

$$\frac{K_t}{N_t} = \frac{\theta\ell - \underline{c}}{\delta} = 1.6 \tag{39}$$

for the same depreciation rate of $\delta = 0.05$. When $N_t = 500$, capital stock this time has to be $K_t = 800$ at the steady state. Hence, a steady state equilibrium is summarized as $(K^*, N^*, Y^*, C^*, S^*, I^*) = (800, 500, 120, 80, 40, 40)$. When $K_t > 400$, the capital stock tends to converge to this steady state of $K^* = 800$ from below or above. In this sense, this steady state is said to be *stable*.

Neoclassical Golden Rule of Capital Accumulation

The above analysis of steady state of capital stock is attained against the steady state of population. How are these two steady state equilibria modified when population changes? To figure out these relations analytically, let us introduce a neo-classical concept of *per capita capital stock*, which is defined as $k_t = K_t/N_t$. And let $n(= \alpha - \beta)$ be a net growth rate of population. Then the above eight equations, except for the non-renewable resources, are very compactly reduced to a single equation. To do so, the equation (18) is first rewritten as

$$\frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t} = \frac{K_t}{N_t} + \left(\frac{Y_t}{N_t} - \frac{C_t}{N_t}\right) - \delta\frac{K_t}{N_t} \tag{40}$$

A simple calculation, then, results in the following capital growth equation.

$$k_{t+1} = k_t + \frac{1}{1+n}\left(Min\{\frac{1}{v}k_t,\ \theta\ell\} - \underline{c} - (n+\delta)k_t\right) \tag{41}$$

A steady state equilibrium is obtained at $k_{t+1} = k_t$, which in turn yields two equilibrium levels of per capita capital.

For a smaller level of equilibrium we have

$$\underline{k} = \frac{\underline{c}}{\frac{1}{v} - (n+\delta)} \tag{42}$$

This corresponds to the previous case in which output is constrained by capital stock (38). For a larger level of equilibrium we have

$$k^* = \frac{\theta\ell - \underline{c}}{n+\delta} \tag{43}$$

This corresponds to the previous case in which output is constrained by the number of workers (39). Note that even at a steady state of per capita equilibrium, population is allowed to grow at a net growth rate of $n > 0$.

It is easily shown that $\underline{k}$ is an unstable state of equilibrium, since $k_{t+1} < k_t$ for $k_t < \underline{k}$, and $k_{t+1} > k_t$ for $\underline{k} < k_t < k^*$. Thus, per capita capital k_t is, once displaced with the equilibrium, shown to decrease toward zero or converge to k^*. Meanwhile, k^* is a stable state of equilibrium, since $k_{t+1} < k_t$ for $k_t > k^*$.

$$k_t \longrightarrow \begin{cases} 0, & \text{if } k_t < \underline{k} \\ k^*, & \text{if } k_t > \underline{k} \end{cases} \tag{44}$$

Let us examine the stability of per capita capital numerically by allowing the economy to grow out of the initial steady state equilibrium. Depreciation and birth rates are now reset to $(\delta, \alpha) = (0.02, 0.03)$ so that both capital stock and population are allowed to start growing. It is then calculated that $\underline{k} = 0.7619$ and $k^* = 2$. Since the initial population is 500, an unstable equilibrium level of initial capital stock is obtained as $K_{2001} = \underline{k}N_{2001} = 380.95$. This means that if the initial capital stock is less than this amount, per capita capital stock tends to diminish toward zero, and the economy will get stuck eventually. Figure 9 numerically illustrates that when $K_{2001} = 380$ (and $k_{2001} = 0.76$) per capita capital decreases to $k_{2100} = 0.0296$, and eventually to zero.

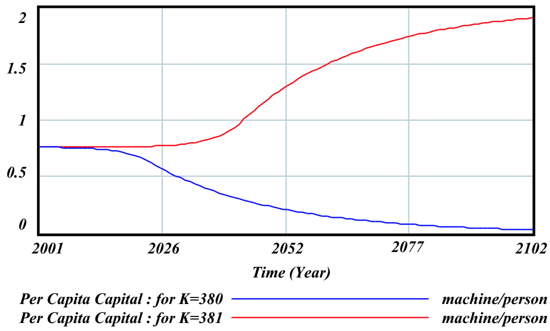

Figure 9: Golden Rule of Capital Accumulation

On the other hand, if the initial capital stock is greater than this amount, per capita capital tends to converge towards a so-called *golden-rule level of capital* in neoclassical growth theory [1]. Figure 9 also illustrates that when $K_{2001} = 381$(and $k_{2001} = 0.762$) per capita capital increases to $k_{2100} = 1.898$, and eventually converges to a golden rule level of capital: $k^* = 2$.

It is interesting to know that only one unit difference of capital stock in our numerical example will result in a big difference in its growth path. When the initial capital stock is $K_{2001} = 380$, the economy will be destined to be trapped forever to a stagnant state, while an additional unit of capital stock will drive the economy up to its prosperity. The importance of an initial level of capital stock for an economic development is a well recognized feature in development economics. To circumvent the situation of this economic trap, the initial capital stock is set so far to $K_{2001} = 400$ in our numerical example.

From now on let us reset the initial value of capital stock, without losing generality, at its critical value of $K_{2001} = 381$. Figure 10 illustrates how capital stock, population, output, consumption and investment grow simultaneously. Population grows at 2%, so does minimum standards of consumption regardless of the growth of capital stock and output level. Output is first constrained by the availability of capital stock, then from the year 2042, it is constrained by the availability of workers, which is in turn constrained by the population growth. Thus, the economy continues to grow at an increasing rate as the capital stock grows up to the year 2042 (from 2% to 5%), then it grows at a constant rate of population growth of 2% . This is why there are some bumps on the output and investment growth paths around 2042.

Even at this growth rate of population, output level is still maintained at a higher level than minimum standards of consumption so that social reproducibility is constantly sustained. Eventually, per capital capital growth will converge to a steady state of k^*, showing a long-run stability of capital accumulation. This is what is meant by a neoclassical economic growth of golden rule: a very elegant and optimistic theory of economic growth!

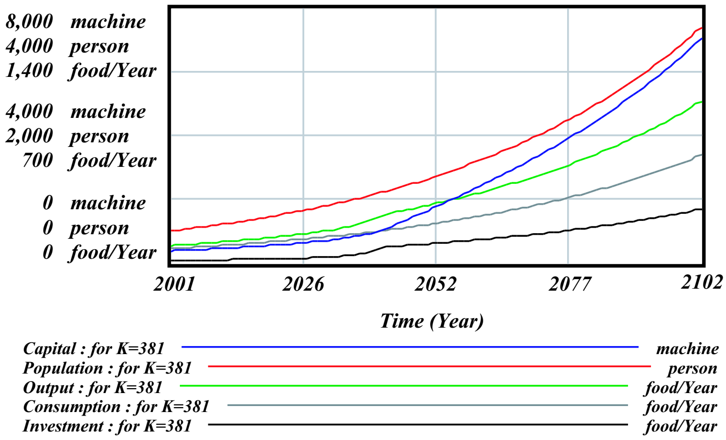

Figure 10: Golden Rule of Economic Growth

Can such a growth be sustainable in the long run, indeed? The answer would be yes as long as non-renewable resources are disregarded and left out of the model. Remembering, however, that neoclassical concept of a steady state allows a constant growth rate of 2% , and the economy still keeps growing, depleting non-renewable resources, the answer would be absolutely no. In fact, non-renewable resources will be totally depleted in the year 2077 in our numerical example and become negative for the next year; that is, $R_{2078} = -13.31$. Even so, neoclassical growth theory keeps silent about this point, giving the impression that our macroeconomy can continue to grow and be stable in the long run.

Feedback Loop for Non-Renewable Resource Availability

Availability of non-renewable resources is now taken into consideration. To make non-renewable resources available for future generations, let us introduce a similar feedback mechanism as implemented in the previous section of physical reproducibility; that is, as the non-renewable resources continue to be depleted, productivity becomes worsened, and accordingly output is curbed, resulting in preserving non-renewable resources. Two constants in the production function (37) could influence productivity separately; that is, capital-output ratio v and output-labor ratio ℓ. Instead of the two constants being affected separately, we introduce a table function that affects output level directly such that

$$Y_t = \text{Productivity}\left(\frac{R_{t+1}}{R_{initial}}\right)\text{Min}\{\frac{1}{v}K_t,\ \ell L_t\} \quad \text{(Production Feedback)} \quad (45)$$

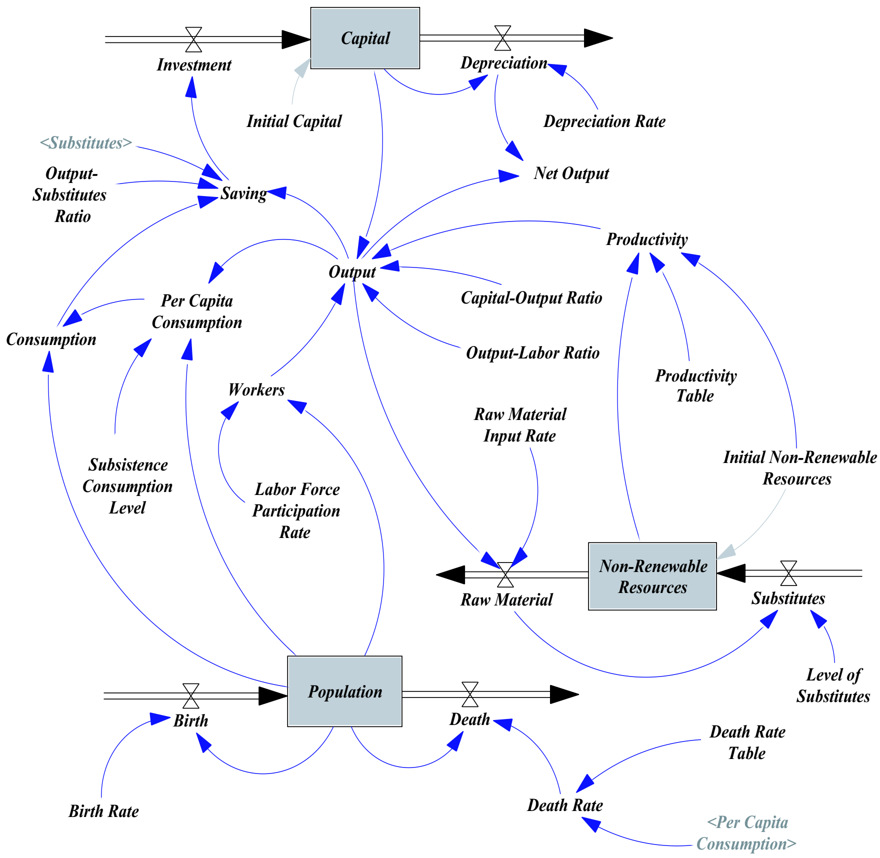

Figure 11: Social Reproducibility Feedback Model

The table function of productivity is defined in Table 7. It is assumed that productivity is not affected until non-renewable resources are depleted up to 40% , beyond which, then, it begins to decrease gradually. Figure 11 illustrates a revised feedback loop for non-renewable resources.

As expected by the introduction of a productivity feedback loop, Figure 12 shows how growth paths of capital stock and net output (output less depreciation) are curved as non-renewable resources continue to be depleted. In this way non-renewable resources are to be preserved.

Feedback Loop for Social Reproducibility

Figure 12 also illustrates that population continues to increase exponentially at a net growth rate of 2% , so does a minimum amount of consumption for

Table 7: A Table Function of Productivity

$R_{t+1}/R_{initial}$	0	0.1	0.2	0.3	0.4	0.5	0.6 - 1
Productivity	0	0.1	0.2	0.4	0.6	0.8	1

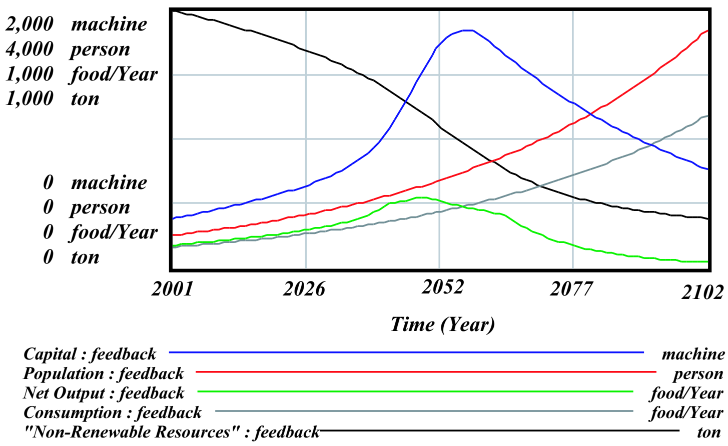

Figure 12: Golden Rule with Non-Renewable Resource Feedback

maintaining a per-capita standards of wholesome and cultured living: $C_t = \underline{c}N_t$. Since net output is curved by a negative feedback loop of non-renewable resources, social reproducibility condition (33) will be eventually violated, and a portion of the population might be forced to be starved to death.

The violation of social reproducibility implies

$$Y_t - D_t - \underline{c}N_t < 0. \tag{46}$$

In our numerical example, this occurs in the year 2057 when $C_{2057} = 242.49$ and $Y_{2057} - D_{2057} = 234.33$, so that consumption exceeds net output by the amount of 8.16 as roughly illustrated in the Figure 12. The violation of social reproducibility implies that a smaller amount of net output has to be shared among people, forcing their level of living standards to be reduced. How far can such a per capita consumption be lowered? For maintaining physical reproducibility, it is desirable to keep its level at which per capita consumption is equal to per capita net output. It would be imaginable, however, that starving people would eat up everything available out of the output, including the reserved amount of capital stock for depreciation.

Reflecting the situation of such food shortage, per capita consumption has to be recalculated as follows[4]:

[4]On the other hand, for maintaining the physical reproducibility, the equation of per capita

$$\text{Per Capita Consumption} = \text{Min}\left(\underline{c},\ \frac{Y_t}{N_t}\right). \tag{48}$$

This formula enables per capita consumption to be lowered from the level of $\underline{c} = 0.16$. A decrease in per capita consumption may increase a death rate due to food shortage. This could happen unevenly among weaker people and children, or among countries whose economy is not wealthy enough to buy food, or among countries that are politically weaker and can be neglected. A table function of death rate in Table 8 is created to reflect such imaginable situations. For instance, whenever a per capita consumption is reduced by half from the original minimum standards, a death rate is assumed to jump to 5% from 1% . In this way, a negative feedback loop of social reproducibility is completed. Figure 11 illustrates a revised feedback version of social reproducibility model.

Table 8: A Table Function of Death Rate

Per Capita Consumption	$\underline{c} = 0.16$	0.14	0.12	0.1	0.08	0.06	0.04
Death Rate β	0.01	0.015	0.02	0.03	0.05	0.07	0.1

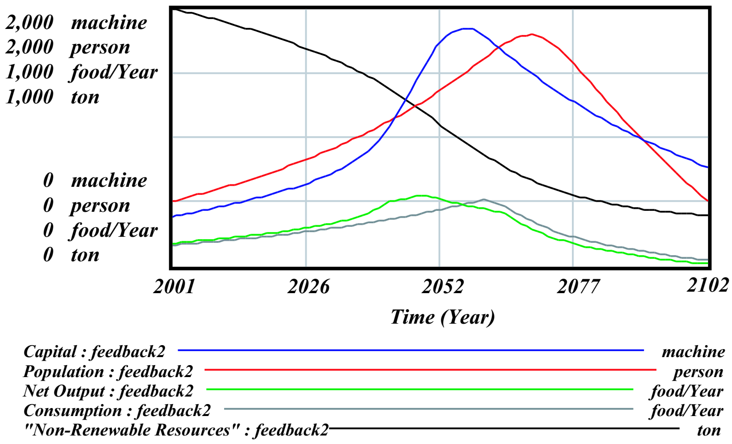

Figure 13: Growth Paths with Social Reproducibility Feedback

Calculated from the revised feedback model, Figure 13 illustrates revised growth paths, reflecting the effect of the feedback loop relation to the death

---

consumption has to be changed to the following:

$$\text{Per Capita Consumption} = \text{Min}\left(\underline{c},\ \frac{Y_t - D_t}{N_t}\right). \tag{47}$$

rate. The amount of consumption exceeds net output during the year 2056, and accordingly capital stock begins to decay. The difference between consumption and net output is the amount of capital depreciation that is allowed to be consumed by hungry people.

A century later, output level becomes only one quarter of its initial level; that is, $Y_{2101} = 25.71$ from $Y_{2001} = 95.25$. Population is almost pulled back to its original level of $N_{2001} = 500$; that is, it increases to its peak at $N_{2069} = 1,793$, then begins to decline to $N_{2101} = 541.51$. Per capita consumption level has been maintained at $\underline{c} = 0.16$ until the year 2059, then begins to decline to the level of 0.0474 in the year 2101 (a 70% decrease !), and the death rate jumps up to almost 10 %. In this way, all economic activities will be trapped. Is there a way to escape from this economic trap?

Substitutes for Non-Renewable Resources

The economic trap mentioned above is basically caused by a diminishing availability of non-renewable resources. To see the effect, let us modify the equation of non-renewable resource depletion (23) so that it allows an inflow of substitutes for non-renewable resources. Let SU_t be an inflow amount of non-renewable

Table 9: Unknown Variable and Constant Added (5)

New Variable	SU_t	Non-Renewable Substitutes	ton/year
New Constant	ν	Level of Substitutes ($0 \leq \nu < 1$)	dimensionless
	ρ	Output-Substitutes Ratio (=1)	food/ton

substitutes, measured by a unit of ton/year, that can be added to the stock of non-renewable resources, and ν be a level of the substitutes such that $0 \leq \nu < 1$. Then the equation (23) is replaced with the following:

$$R_{t+1} = R_t + SU_t - \Delta R_t \quad \text{(Non-Renewable Resource Depletion)} \quad (49)$$

$$SU_t = \nu \Delta R_t \quad \text{(Substitutes for Non-Renewable Input)} \quad (50)$$

Or, combining these two, we have

$$R_{t+1} = R_t - (1 - \nu)\Delta R_t \quad \text{(Non-Renewable Resource Depletion)} \quad (51)$$

Where do the substitutes come from? For simplicity it is assumed that they are converted from the output by a factor of output-substitutes ratio. Saving function (4) then has to be revised as follows:

$$S_t = Y_t - C_t - \rho SU_t \quad (52)$$

Figure 11 illustrates the SD modeling implementation of the substitutes for non-renewable resources.

Several simulations are done, under such circumstances, to attain the growth paths of the golden rule of capital accumulation illustrated in Figure 10. It turned out that at least 400 unit machines of initial capital stock are needed, for a 80% level of substitutes, to drive an economic growth initially. So the initial capital stock is reset again to $K_{2001} = 400$. Even so, if a level of substitutes is set higher than 80%, the economy again turns out to be trapped. This is a little bit surprising result, because a higher rate of substitutes is supposed to preserve the non-renewable resources. A moment of thought clarifies the reason. A higher level of substitutes subtracts more portion of output, and capital accumulation begins to decline with less saving and investment.

On the other hand, a lower level of substitutes depletes non-renewable resources faster, reducing productivity and output. Again, the economy is trapped somewhere in the middle, and cannot attain the golden rule growth paths over the entire 21st century. Only when a level of substitutes is 80%, the economy can recover from the economic trap that is caused by a negative feedback loop of non-renewable resources as illustrated in Figure 13, and once again begin to attain the growth paths of golden rule for the entire 21st century. Figure 14 illustrates such golden rule growth paths up around the turn of the 21st century.

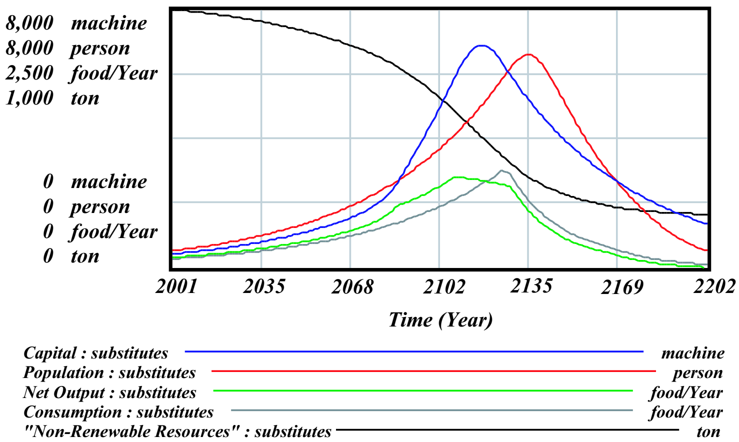

Figure 14: Social Reproducibility Economic Trap in the Long Run

However, this is nothing but postponing a problem of economic trap to the 22nd century. Moreover, there will be no way to escape from the economic trap in the 22nd century, no matter how Output-Substitutes Ratio is reduced and saving and investment is restored. Figure 14 shows exactly the same structure as in the Growth Paths with Social Reproducibility Feedback in Figure 13, except that a time scale is elongated over two centuries in the present case. Substitutes of non-renewable resources cannot be an economic savior in the long run.

DISCUSSIONS FOR THE SECTION:

(a) Refer to microeconomics textbooks and discuss how production activities can be described under different production functions such as the equations (36) and (37). Advanced reader may replace the production function (37) with a Cobb-Douglas production function (36) in the Social Reproducibility Model (Figure 13) and continue simulation analysis similarly.

(b) Derive the capital growth equation (41), and, by using the equation, show how k_t converges to two different points. It will be helpful for understanding this convergence movement intuitively if you draw the diagram of the items in the parenthesis of the equation (41) separately.

(c) With the introduction of the Table Function of Death Rate in Table 8, it is briefly assumed that a food shortage will eventually increase a death rate. Discuss in reality how food shortage will affect people and countries world wide unevenly.

5 Ecological Reproducibility

Production and consumption activities as well as capital accumulation formalized above produce as by-products consumer garbage GC_t, industrial wastes GY_t and capital depreciation dumping GK_t. These by-products are in turn dumped into the earth or scattered around atmosphere and accumulated as an artificial environmental stock called sink SK_{t+1}. Some portion of the sink will be naturally regenerated (or recycled) and made available as renewable resource stock that is called source SR_{t+1}. As a typical example, we can refer to photosynthesis processes in which tropical forests and trees grow by taking carbon dioxides (industrial wastes) as inputs and producing oxygen as by-product output.

These three dumping processes together with an extracting process of non-renewable resources now form an entire global environment Env, consisting of the earth's sink and source. Hence, the formation of the entire global environment might be appropriately considered as an ecological reproduction process which is symbolically illustrated as:

$$(\ominus \Delta R_t \oplus GC_t \oplus GY_t \oplus GK_t) \Longrightarrow Env(SK_{t+1} \rightarrow SR_{t+1}). \qquad (53)$$

To describe such an ecological reproduction process, we need to add the following seven equations.

$$SK_{t+1} \equiv SK_t + \Delta SK_t \qquad \text{(Accumulation of Sink)} \qquad (54)$$
$$\Delta SK_t = GC_t + GY_t + Gk_t - (\epsilon + \mu)SK_t \qquad \text{(Net Change in Sink)} \qquad (55)$$
$$GC_t = \gamma_c C_t \qquad \text{(Consumer Garbage)} \qquad (56)$$
$$GY_t = \gamma_y Y_t \qquad \text{(Industrial Wastes)} \qquad (57)$$
$$GK_t = \gamma_k D_t \qquad \text{(Depreciation Dumping)} \qquad (58)$$
$$SR_{t+1} \equiv SR_t + \Delta SR_t \qquad \text{(Accumulation of Source)} \qquad (59)$$
$$\Delta SR_t = (\epsilon + \mu)SK_t - \lambda_1 Y_t \qquad \text{(Net Change in Source)} \qquad (60)$$

Table 10: Unknown Variables and Constants Added (6)

New Variables	SK_{t+1}	Sink	source
	SR_{t+1}	Source	source
	GC_t	Consumer Garbage	source/year
	GY_t	Industrial Wastes	source/year
	GK_t	Capital Depreciation Dumping	source/year
New Constants	ϵ	Natural Rate of Regeneration (= 0.15)	1/year
	μ	Recycling Rate (= 0.05)	1/year
	λ_1	Renewable Raw Material Input Rate (= 0.6)	source/food
	γ_c	Garbage Rate(= 0.5)	source/food
	γ_y	Industrial Wastes Rate (= 0.1)	source/food
	γ_k	Depreciation Dumping Rate (= 0.5)	source/machine
Initial values	SK_t	Initial Sink (=300)	source
	SR_t	Initial Source (=3,000)	source

In order for an ecological reproduction process to continue, total amount of consumer garbage, industrial wastes and capital depreciation dumping have to be less than the earth's ecological capacity to absorb and dissolve the sink, and those newly regenerated source have to add enough amount to renewable source for continued production activities. Otherwise, the amount of sink begins to accumulate, and the accumulated sink will eventually cause the environment to collapse, or renewable source will be completely depleted. Therefore, for a sustainable ecological reproducibility, the following two conditions have to be met.

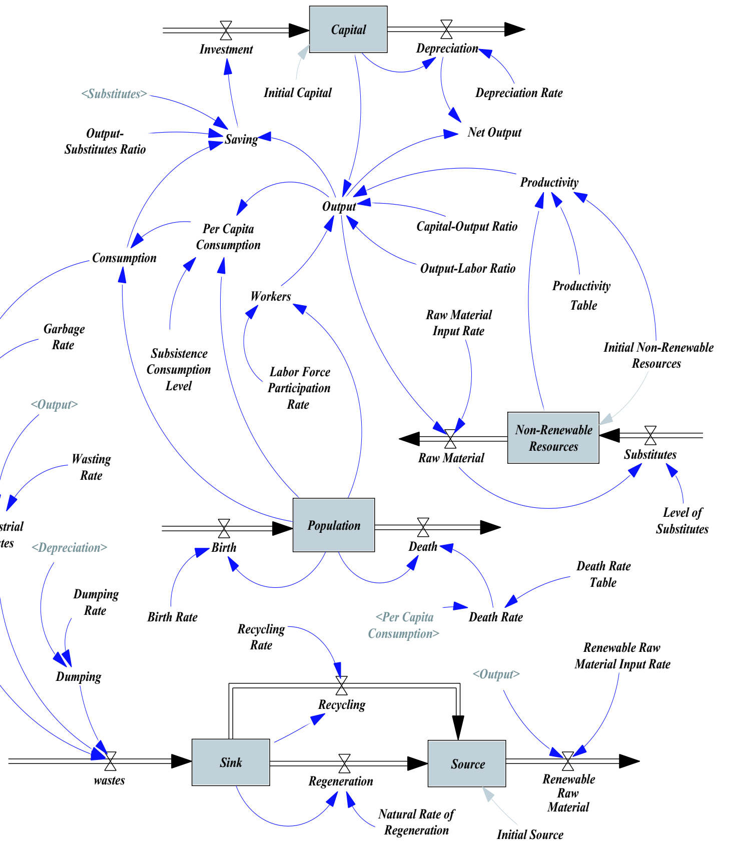

Figure 15: Ecological Reproducibility Model

$$\sum_{t=2001}^{\infty} (GC_t + GY_t + GK_t) \leq \epsilon \sum_{t=2001}^{\infty} SK_{t+1} \quad \text{[Ecological Reproducibility]} \tag{61}$$

$$SR_{t+1} > 0,\ t = 2001,\ \cdots \quad \text{[Renewable Source Availability]} \tag{62}$$

Fortunately, the ecological reproducibility of recycling sink into source and restoring the original ecological shape has been built in the earth as a self-regulatory mechanism of Gaia [6]. Consumer garbage, industrial wastes and capital depreciation dumping have been taken care of and disintegrated by a natural reproduction process, and the environment so far seems to have continued to restore itself to a certain degree. Therefore, a sustainable development might be possible for the time being so long as the accumulated sink which the ecological reproduction process fails to disintegrate does not reach the environmental capacity of regeneration.

As production and consumption activities expand exponentially, however, such environmental sink also continues to accumulate exponentially. And naturally built-in ecological reproducibility of Gaia eventually begins to fail to regenerate the sink so that a portion of the sink will be left unprocessed Eventually, an environmental catastrophe occurs, and the earth might become uninhabitable for many living species, including human beings. In fact, many environment scientists warn us that such a catastrophe has already begun. For instance, see [2].

Accordingly, to be able to stay within a limit to ecological reproducibility, first of all, the total amount of environmental sink has to be directly regulated within an environmentally regenerating capacity. Second, new development of recycling-oriented products has to be encouraged so that the amount of environmental sink is reduced at every stage of reproduction and consumption process. Third, hazardous and toxic wastes which are not naturally disposed of have to be chemically processed and recycled safely at all costs. Then, the equation of ecological reproducibility (61) is expanded as follows:

$$\sum_{t=2001}^{\infty} (GC_t + GY_t + GK_t) \leq (\epsilon + \mu) \sum_{t=2001}^{\infty} SK_{t+1} \quad \text{[Recycling of Sink]} \qquad (63)$$

A Steady State Equilibrium

A steady state equilibrium of the ecological reproducibility is attained at $SK_{t+1} = SK_t$ and $SR_{t+1} = SR_t$; that is, $\Delta SK_t = \Delta SR_t = 0$. From the above equations of ecological reproducibility this implies

$$GC_t + GY_t + Gk_t = (\epsilon + \mu)SK_t = \lambda_1 Y_t \qquad (64)$$

A steady state of capital accumulation is already obtained under the section of physical reproducibility. Using the same numerical values of that steady state, and constant values assigned in Table 10, we have

$$GC_t + GY_t + Gk_t = 0.5 \cdot 80 + 0.1 \cdot 100 + 0.5 \cdot 20 = 60. \qquad (65)$$

$$(\epsilon + \mu)SK_t = (0.15 + 0.05)300 = 60. \qquad (66)$$

$$\lambda_1 Y_t = 0.6 \cdot 100 = 60. \qquad (67)$$

A steady state of population growth is attained when rates of birth and death are equal as already shown under the section of social reproducibility. Hence, a steady sate of ecological reproducibility is shown to exist and our model of the ecological reproducibility becomes consistent. However, this is no longer true if non-renewable resources are considered explicitly. Figure 16 illustrates that

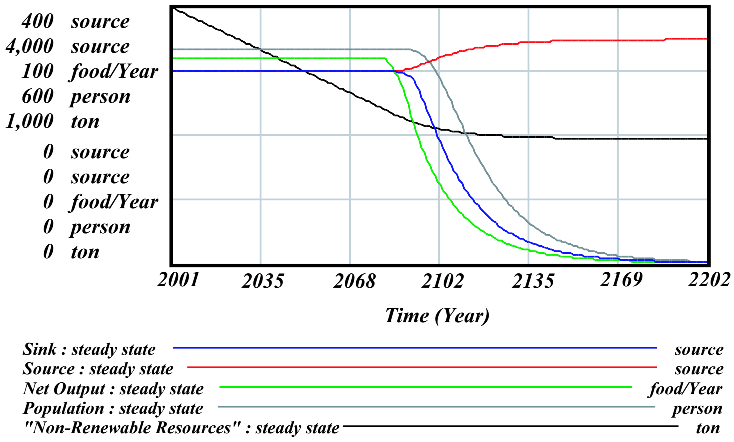

Figure 16: A Steady State of Ecological Reproducibility

an ecological steady state equilibrium is sustained almost throughout the 21st century until net output starts decreasing in the year 2082. This decrease in the net output is caused by a diminishing productivity, which is in turn caused by the depletion of non-renewable resources. Accordingly, per capita consumption decreases and a death rate increases, resulting in a decline of population growth that begins to start in the year 2091, a decade later. Hence, an ecological steady state equilibrium becomes impossible in the long run if non-renewable resources are taken into consideration.

Simulations for a Sustainable Growth

When a depreciation rate and a birth rate are reset at the original values; that is $\delta = 0.02$ and $\alpha = 0.03$, respectively, the economy begins to grow. However, this growth paths are eventually curbed by a decrease in net output, and declining population that follows it, as illustrated in Figure 17.

To maintain the growth paths, a level of substitutes might be again set to be 80% as in the previous section. Then the net output and population once again continue to grow for the entire 21st century. However, this sustained growth paths begin to cause ecological unsustainability, since the amount of sink continues to accumulate and source is completely depleted in the year 2090 as illustrated in Figure 18.

Eventually some negative feedback loops might emerge to prevent such environmental catastrophes. For instance, the over-accumulation of the sink such as chemical wastes will surely affect human health, and as a result a birth rate will be reduced and a death rate will be increased : an emergence of new feedback loops from sink to birth and death rates. Since a feedback loop of the death

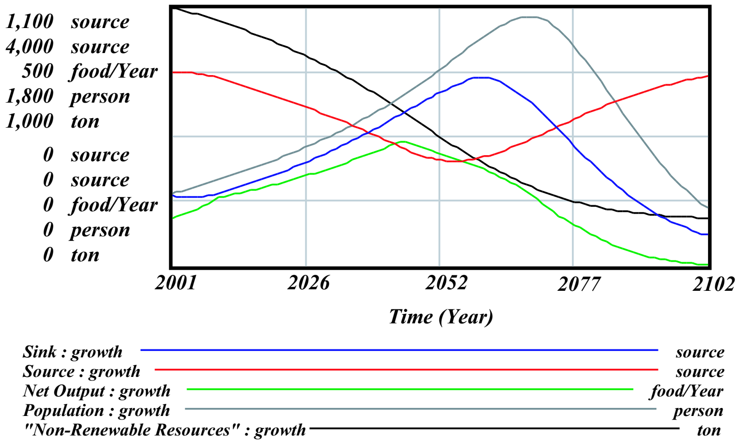

Figure 17: Growth Paths of Ecological Reproducibility

rate is introduced in Table 8, let us consider only a feedback loop of the birth rate as the table function in Table 11.

Table 11: A Table Function of Birth Rate

$SK_{t+1}/SK_{initial}$	0	0.5	1	2	3	4	5	10
Birth Rate α	0.05	0.04	0.03	0.02	0.01	0.008	0.005	0.003

The birth rate is here assumed to decrease from the initial value of 3% , as the amount of sink continues to increase. When the amount of sink triples, it is assumed to become the same as the initial death rate of 1% . In this way, a change in population as a whole is assumed to depend on both the interplay between the accumulated level of sink and birth rate, and the availability of per capita consumption and death rate.

Meanwhile, as renewable source continue to be depleted, output will be curbed as in the case of the depletion of non-renewable resources: a feedback loop from source to output. Let us consider such feedback loop by introducing a second table function of productivity, called an ecological productivity or productivity2, as defined in Table 12.

Table 12: A Table Function of Ecological Productivity

$SR_{t+1}/SR_{initial}$	0	0.1	0.2	0.3	0.4	0.5	0.6 - 1	2
Productivity2	0	0.1	0.2	0.4	0.6	0.8	1	1.2

The table function is assumed similarly as in Table 7 such that the productivity is not affected until renewable source is depleted up to 40%, beyond which,

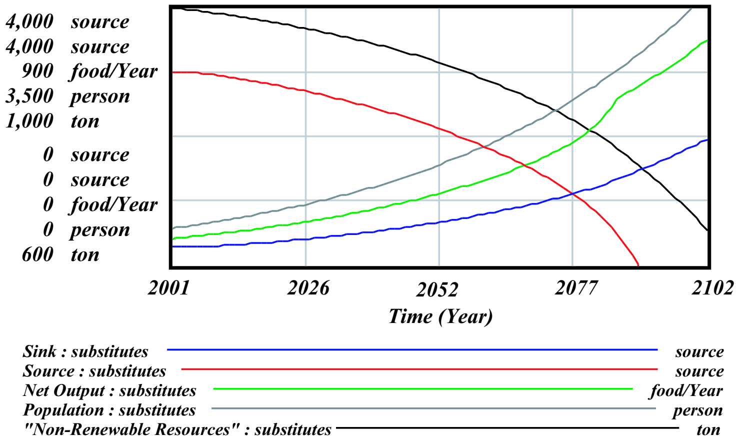

Figure 18: Growth Paths with Non-Renewable Substitutes

then, it begins to decrease gradually. Since renewable source could increase beyond the initial amount due to recycling and natural regeneration, productivity is also assumed to increase by 20% in that case.

The equation of the Production Feedback (45) may now be redefined as

$$Y_t = \text{Productivity}\left(\frac{R_{t+1}}{R_{initial}}\right)\text{Productivity2}\left(\frac{SR_{t+1}}{SR_{initial}}\right)\text{Min}\{\frac{1}{v}K_t,\ \ell L_t\} \quad (68)$$

This implies that output level is affected by the remaining ratios of both non-renewable resources and renewable source.

By newly introducing these two feedback loops, ecological reproducibility could be restored by avoiding the problems of the over-accumulation of the sink and the depletion of the renewable source and non-renewable resources. To see this possibility, Monte Carlo or multivariate sensitivity simulations are done over two constants; that is, a level of substitutes for non-renewable resources and an output-substitutes ratio. To be specific, each constant value is randomly chosen between 0 and 1 according to the Random Uniform Distribution, and simulations are repeated 200 times. Figure 19 shows a simulation result of confidence bounds by the growth paths of population. A line in the middle of the graph indicates a mean value of the population, which indicates that population growth cannot be sustained as a mean value. In fact, the growth paths of population are shown to be unsustainable within at least 75% of confidence bounds. In other words, even with the introduction of ecological feedback loops, it is very hard to randomly find a sustainable path over the 22nd century. The Monte Carlo simulation shows that if we could find it, we would be very lucky!

One such ecologically sustainable growth path over the next two centuries

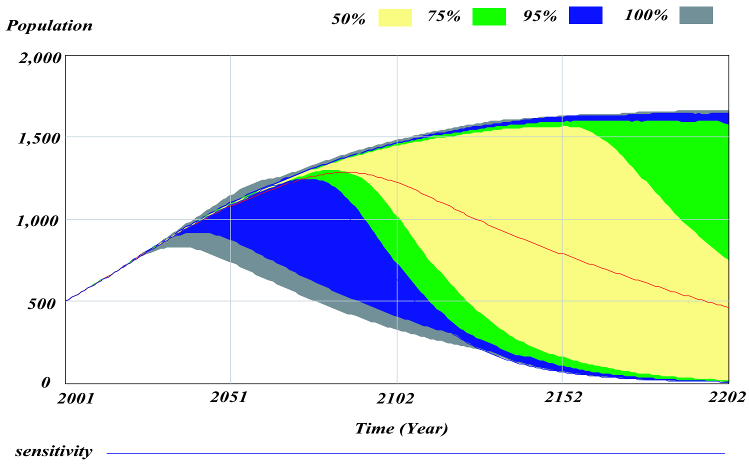

Figure 19: Ecological Sensitivity Simulation of Population

could be luckily attained at the 80% level of substitutes for non-renewable resources and the output-substitutes ratio of 20% as illustrated in Figure 20. Even so, it would be burdensome to attain such constant values in reality. How can we produce 80% substitutes of non-renewable resources such as oil and other fossil energies? How can we make such technology of substitutes more efficient, by lowering the output-substitutes ratio, so that the amount of output is not excessively eaten up for the production of substitutes? In our numerical example, to attain the above ecological sustainability, the ratio has to be lowered to 0.2 from the initial value of 1 that is assumed in the previous section of social reproducibility.

Even if this lucky sustainable path is found, there will be no way of escaping from the economic trap as illustrated in Figure 14. The economic trap will eventually emerge in the 23rd century so long as non-renewable resources continue to be depleted and production of substitutes continue to eat up the amount of output, leaving less and less amount for consumption, saving and investment. In fact, as soon as our numerical simulation in Figure 20 is extended into the 23rd century, such an economic trap will begin to emerge on the horizon!

DISCUSSIONS FOR THE SECTION:

(a) As a fundamental cause of contaminating the earth, three dumping processes of production and consumption activities are conceptually classified: consumer garbage, industrial wastes, and capital depreciation dumping. Consider as many examples of such dumping processes as passible, and discuss whether such a classification of dumping processes is appropriate or not in building an ecological sustainable model.

(b) A feedback loop from sink to the death rate is not introduced in the above

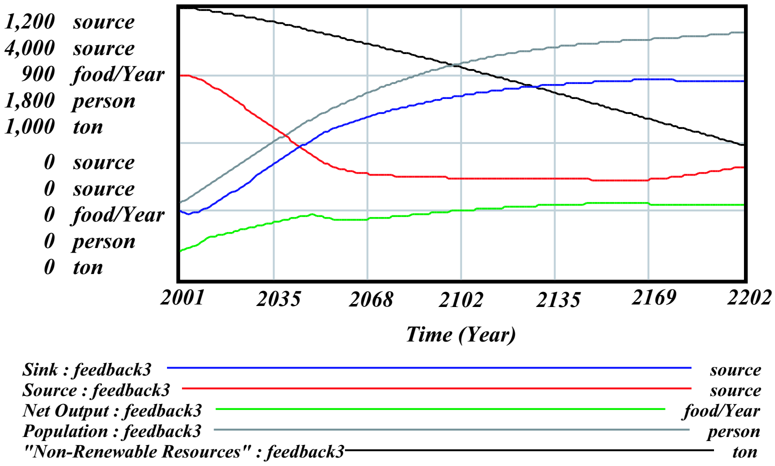

Figure 20: Growth Paths with Ecological Feedback

ecological feedback simulation. Discuss how accumulated hazardous wastes and chemical toxics affect both birth and death rates by referencing books such as [2]. Then, try to formalize more comprehensive way to introduce two feedback loops in the Ecological Reproducibility Model.

(c) A dismal conclusion from the simulation of the ecological feedback model would be no sustainable growth path is possible in the long run as long as our production and consumption activities are dependent on non-renewable resources. Do you agree ? Discuss how long could be *the long run*, and possible policies and our living styles to elongate it.

Conclusion

We have constructed a system dynamics model of long-term sustainability step-by-step from a simple macroeconomic growth model. Yet our simulations for sustainability are far from comprehensive, and numerical data used in the model are not the real one. Hence, there could be many ways to expand or revise the model for further consideration of a long-term sustainability within a region, a country, or globally. Whichever sustainability model being constructed, however, the logic developed in our model will remain worth considered. In this sense, the system dynamics model developed here would become genetic for sustainability modeling. One of the conclusions from the model is that no long-term sustainability is possible under the usage of non-renewable resources, and it will remain a challenging issue for any sustainability model yet to be developed.

Appendix

A complete list of the Vensim equations for Ecological Reproducibility Model in Figure 15 is provided below as the reader's reference of system dynamics modeling. They are arranged in the order as variable and constant names appear so that the reader can easily follow the equations as if he or she is reading a story from the system dynamics model.

```
Capital= INTEG (Investment - Depreciation, Initial Capital)
     Unit: machine
Initial Capital = 400
     Unit: machine
Investment = S to I Conversion * Saving
     Unit: machine/Year
S to I Conversion = 1  (This constant is hidden in the model)
     Unit: machine / food
Saving = MAX(Output - Consumption -
                       ‘‘Output-Substitutes Ratio’’ * Substitutes, 0)
     Unit: food/Year
‘‘Output-Substitutes Ratio’’ = 1
     Unit: food/ton
Depreciation = Depreciation Rate * Capital
     Unit: machine/Year
Depreciation Rate = 0.02
     Unit: 1/Year
Output = Productivity * MIN(Capital / "Capital-Output Ratio",
              "Output-Labor Ratio" * Workers
     Unit: food / Year
Net Output = Output - Depreciation /S to I Conversion
     Unit: food/Year
Productivity = Productivity Table ("Non-Renewable Resources" /
                       "Initial Non-Renewable Resources")
     Unit: Dmnl
Productivity Table([(0,0)-(1,1)],(0,0),(0.1,0.1),(0.2,0.2),
                 (0.3,0.4),(0.4,0.6),(0.5,0.8),(0.6,1),(1,1))
     Unit: Dmnl
"Capital-Output Ratio" = 4
     Unit: machine / (food / Year)
"Output-Labor Ratio" = 0.4
     Unit: (food / Year) / person
Workers = Labor Force Participation Rate * Population
     Unit: person
Labor Force Participation Rate = 0.6
     Unit: person / person
Population= INTEG (Birth - Death, 500)
     Unit: person
Birth = Birth Rate * Population
```

```
     Unit: person / Year
Birth Rate = 0.03
     Unit: (person / person) /Year
Death = Death Rate * Population
     Unit: person/Year
Death Rate = Death Rate Table ( Per Capita Consumption )
     Unit: 1/Year
Death Rate Table([(0.04,0)-(0.16,0.1)],(0.04,0.1),(0.06,0.07),
          (0.08,0.05),(0.1,0.03),(0.12,0.02),(0.14,0.015),(0.16,0.01))
     Unit: 1/Year
Consumption = Per Capita Consumption * Population
     Unit: food/Year
Per Capita Consumption = MIN(Minimum Standards of Consumption,
                                         Output/Population)
     Unit: (food/Year) / person
Subsistence Consumption Level = 0.16
     Unit: (food / Year ) / person
"Non-Renewable Resources"= INTEG ( Substitutes - Raw Material,
                          "Initial Non-Renewable Resources")
     Unit: ton
"Initial Non-Renewable Resources" = 1000
     Unit: ton
Substitutes = Level of Substitutes * Raw Material
     Unit: ton/Year
Level of Substitutes = 0
     Unit: Dmnl
Raw Material = Raw Material Input Rate * Output
     Unit: ton / Year
Raw Material Input Rate = 0.05
     Unit: ton / food
Sink= INTEG ( wastes - Regeneration - Recycling, 300)
     Unit: source
wastes = Consumer Garbage + Industrial Wastes + Dumping
     Unit: source/Year
Consumer Garbage = Garbage Rate * Consumption
     Unit: source/Year
Garbage Rate = 0.5
     Unit: source / food
Industrial Wastes = Wasting Rate * Output
     Unit: source/Year
Wasting Rate = 0.1
     Unit: source/food
Dumping = Dumping Rate * Depreciation
     Unit: source/Year
Dumping Rate = 0.5
     Unit: source / machine
```

```
Regeneration = Natural Rate of Regeneration * Sink
     Unit: source / Year
Natural Rate of Regeneration = 0.15
     Unit: 1 / Year
Recycling = Recycling Rate * Sink
     Unit: source/Year
Recycling Rate = 0.05
     Unit: 1/Year
Source= INTEG (Regeneration + Recycling - Renewable Raw Material,
                        Initial Source)
     Unit: source
Initial Source = 3000
     Unit: source
Renewable Raw Material = Renewable Raw Material Input Rate * Output
     Unit: source/Year
Renewable Raw Material Input Rate = 0.6
     Unit: source / food
****************
INITIAL TIME  = 2001
     Unit: Year
FINAL TIME  = 2101
     Unit: Year
****************
```

References

[1] Olivier Blanchard. *Macoeconomics*. Prentice Hall, Inc., New York, 1997.

[2] Theo Colborn, Dianne Dumanoski, and John Peterson Myers. *Our Stolen Future*. A Plume/Penguin Book, New York, 1997.

[3] Dennis L. Meadows Donella H. Meadows and Jorgen Randers. *Beyond the Limits*. Chelsea Green Publishing, Post Mills, Vt., 1992.

[4] Andrew Ford. *Modeling the Environment*. Island Press, Washington,D.C., 1999.

[5] Jay W. Forrester. *World Dynamics*. Wright-Allen Press, Inc., Cambridge, Massachusetts, 1971.

[6] J. E. Lovelock. *The Ages of Gaia: A Biography of Our Living Earth*. Bantam Books, New York, 1988.

[7] World Commission on Environment and Development. *Our Common Future*. Oxford University Press, Oxford, 1987.

[8] Kaoru Yamaguchi. *Beyond Walras, Keynes and Marx – Synthesis in Economic Theory Toward a New Social Design*. Peter Lang Publishing, Inc., New York, 1988.

[9] Kaoru Yamaguchi, editor. *Sustainable Global Communities in the Information Age – Visions from Futures Studies*, chapter 5, pages 47 – 65. Adamantine Press Limited, 1997.

[10] Kaoru Yamaguchi. Stock-flow Fundamentals, Dealt Time (DT) and Feedback Loop – from Dynamics to System dynamics. *OSU Journal of Business Administration*, 1(2):57 – 76, 2000.